Using PDE's for Noise Reduction in Time Series

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Abstract:
In this paper, a new method is presented for noise reduction in signal using partial differential equations. In this approach, the signal is initially represented as a matrix**.** Then using singular values of the matrix, noisy data matrix is divided into signal subspace and noise subspace Since singular vectors are the span bases of the matrix, reducing the effect of noise from the singular vectors and using them in reproducing the matrix enhances the information embedded in the matrix. The proposed technique utilizes the Partial Differential Equations (PDEs) for noise attenuation from the singular vector.The enhanced matrix is finally transformed to a time series vector. To evaluate performance of the proposed method, a number of experiments have been performed on both multi-component and FM signals cluttered with noise. The results indicate that the proposed method outperforms the existing approach, in signal de-noising.

Keywords: PDEs, De-Noising, Time Series.

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1. INTRODUCTION

In signal processing and communication systems, applying a robust noise reduction is very important task. Hence, many researchers are investigating to develop a comprehensive noise reduction technique, and many techniques have so far been proposed to attain this goal [Hassanpour, H, dsp.2007], [Rocker, J et al., 2002], [Whipple, G, 1994], [Maris, E, 2003], [Hassanpour, H, ISCAS 2007].

Noise reduction is a very challenging problem. The nature and the characteristics of noise change significantly from application to application. In addition, noise characteristics vary in time. It is therefore very difficult to develop a versatile algorithm that works in diversified environments. Also, the objective of a noise reduction system may depends on the specific context and application. In signal de-noising applications we should consider two points. First, Improving signal to noise ratio (SNR) for

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obtaining clean and readily observable signal. Secondly, preserve the original shape characteristics of the signal.

Currently, there are a number of methods for signal de-noising [Hassanpour, H., dsp.2007], [Rocker, J et al., 2002], [Whipple, G, 1994], [Maris, E, 2003], [Hassanpour, H, ISCAS 2007], [Li, J. and Akagi, M, 2006]. One of the methods which has been widely used in signal de-noising is the Wiener filter. The Wiener filter is always able to reduce noise in a signal. However, the amount of noise reduction is proportionally accompanied by signal degradation. In other words, Wiener filter can be used to reduce noise in a signal if the SNR is high (higher than 4). When SNR in a signal is low, using Wiener filter may just transform the noise from one form to another. Using time-frequency distribution is another method that has been recently introduced for signal de-noising [Hassanpour, H., dsp.2007], [Hassanpour, H., ISCAS 2007]. This method is considerably effective in reducing noise even with low SNR. However, this approach can only be used for bidirectional time-frequency distribution to reduce noise from time-series. This technique is based on the Singular Value Decomposition (SVD) of the matrix associated with the time–frequency representation of the signal. Indeed, in this approach the time-frequency distribution is used as a tool for representing the signal in a matrix. But in this method, a high computational time is required for representing signal in the time-frequency domain.

Recently, time domain based approaches for noise reduction have received a considerable attention among researches [Btocker, J et al., 2002]. These techniques construct a time data matrix, often the Hankel matrix, of the noisy signal. In this paper, the data matrix is divided into signal subspace and noise subspace using the SVD-based approach introduced in [Hassanpour, H., ISCAS 2007]. Then using partial differential equations (PDEs) the noise from the singular vectors (SVs) are reduced and these SVs are used to reconstruct the matrix. This noise-reduced matrix is used to extract the time series, representing the noise-attenuated signal. Using PDEs is one of the methods that have been initially used for image de-noising techniques [Hassanpour, H., ISSPA 2007], [Nadernejad,E. et al., 2007], [Perona, P. and Malik, J, 1990]. Results in this paper show that PDEs are very powerful for image de-noising compared to other exiting approaches. These results prompted us to use PDE for noise reduction in signal. Results in this research indicate that PDEs are suitable for signal de-noising.

The remaining of this paper is organized as follows: section 2. discusses the Wiener filter ; section 3. presents the proposed noise reduction method; section 4. gives the performance evaluation and criteria metrics; section 5. discusses the experimental results and section 6. gives conclusions about the study.

2. WIENER FILTER

This filter has been widely used in signal processing for reducing the noise [Chen, J. et al., IEEE Trans. 2006]. In this method, the noisy signal is passed through a finite impulse response (FIR) filter whose coefficients are estimated by minimizing the mean square error (MSE) between the clean signal and its estimate to restore the desired signal. This filter is one of the most fundamental approaches for noise reduction, which can be formulated in the time and also in the frequency domains.
Assume that a clean signal is corporate with unknown noise:

$$y(n) = x(n) + v(n) \tag{1}$$

Where $x(n)$ is the original signal, $v(n)$ is noise, and $y(n)$ is the time-series noisy signal. Our goal is to estimate $x(n)$ from $y(n)$.

In the time-domain Wiener filter, an estimate of the restored signal $\hat{x}(n)$ is obtained by passing the noisy signal y(n) through a temporal filter:

$$\hat{x}(n) = h^T y(n) \tag{2}$$

Where:

$$h = \left[h_0 h_1 \ldots h_{L-1}\right]^T \tag{3}$$

is an FIR filter of length L. The MSE criterion is then computed as:

$$\varepsilon(h) \overset{\Delta}{=} E\left\{\left[x(n) - \hat{x}(n)\right]^2\right\} = E\left\{\left[x(n) - h^T y(n)\right]^2\right\} \tag{4}$$

where E{.} denotes the mathematical expectation. The objective of noise reduction is to find the optimal *h* that minimizes $\varepsilon(h)$.

In this filter it is assumed that signal and noise have Gaussian distribution with zero mean. Wiener filter has a high capability in reducing Gaussian noises. As mentioned before this filter aims to minimize the total mean square error, therefore it may not consider the local features. Also, this filter can only reduce the noise on signals, if the SNR value is higher than 4. if SNR value is low this filter can only reshape the noise.

3. THE PROPOSED NOISE REDUCTION METHOD

In this paper, it is supposed that the clean signal has been corrupted by an additive white Gaussian noise. For $X_n(i), i=1,\ldots,N$ representing the noisy signal, the Hankel matrix is constructed as follows:

$$H=\begin{bmatrix} X_n(1) & X_n(2) & \cdots & X_n(K) \\ X_n(2) & X_n(3) & & X_n(K+1) \\ \vdots & \vdots & & \vdots \\ X_n(L) & X_n(L+1) & \cdots & X_n(N) \end{bmatrix} \tag{5}$$

The singular value decomposition of matrix H with size $P\times Q$ is of the form:

$$H=U\Sigma V^T \tag{6}$$

Where $U_{P\times r}$ and $V_{r\times Q}$ are orthogonal matrices, and Σ is an $r\times r$ diagonal matrix of singular values with components $\sigma_{ij=0}$ if $i\neq j$ and $\sigma_{ii}>0$. Furthermore, it can be shown that $\sigma_{11}\geq\sigma_{22}\geq\cdots\geq 0$. The columns of the orthonormal matrices U and V are called the left and right SVs respectively.
The subspace separation introduced in [H.Hassanpour, ISCAS 2007] can be briefly expressed as below:

$$H=U\Sigma V^T=\begin{pmatrix} U_s & U_n \end{pmatrix}\begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_n \end{bmatrix}\begin{pmatrix} V_s^T \\ V_n^T \end{pmatrix} \tag{7}$$

$$X_s=U_sU_s^TH=HV_sV_s^T \tag{8}$$

$$W_n=U_nU_n^TH=HV_nV_n^T \tag{9}$$

Where Σ_s and Σ_n represent the clean signal subspace and noise subspace, respectively. As can be seen from equation (6), we must determine a threshold point in the Σ matrix where lower singular values from that point can be categorized as noise subspace and hence should be set to zero [H.Hassanpour, ISCAS 2007]. This threshold point can be determined by calculating derivation of the curve in each point. Our research shows that the noise subspace is mainly related to those singular values that are lower than this threshold point. Thus, we suggest setting these singular values to zero for space division.

By merely filtering the singular values, some noisy data will still be available in the signal subspace. To further enhance information embedded in the Hankel matrix, PDEs are applied to filtering SVs of the signal subspace matrix to reduce the noise effect.

PDEs can reduce the noise like the low-pass filters without shifting effect on the obtained signal.

Partial differential equations have been initially introduced for image noise reduction. One of these equations which have been used in image processing applications is the heat equation. This equation is defined as follows:

$$\frac{\partial I(x,y,t)}{\partial t}=\nabla.(c(x,y,t)\nabla I(x,y,t) \tag{10}$$

Where $I(x,y,t)$ is the noisy image and $c(x,y,t)$ is the influence coefficient.

In this method, gradient in four directions of any pixels are calculated and then their influencing coefficients are obtained to reduce the noise using (10). Then, with a number of iterations, the enhanced image is obtained.

But in the form of a signal, the gradient of each sample is computed using the samples before and after the current sample. Then, the influencing coefficients in each directions of the current sample, forward (c_f) and backward (c_b), are computed as follows:

$$S(x,t+\Delta t)=S(x,t)+\Delta t(d_fc_f+d_bc_b) \tag{11}$$

$$d_f = S(x-\Delta x,t) - S(x,t) \qquad c_f = \frac{1}{1+\left(\frac{d_f}{k}\right)^2}$$
$$d_b = S(x+\Delta x,t) - S(x,t) \qquad c_b = \frac{1}{1+\left(\frac{d_b}{k}\right)^2} \tag{12}$$

In equations (11) and (12), $S(x,t)$ is the noisy signal, in this case it represent SVs. d_f, d_b are gradient in forward and backward direction, c_f, c_b are the corresponding influencing coefficients for each of the directions, k is a constant value between 5 and 100, Δt is a coefficient between 0.1 to 0.3 representing the step of noise reduction in each iteration, and Δx is the sampling rate.

The output vector is again applied to the algorithm at the next iteration to gradually reduce the noise. This process is repeated for the number iterations that lead to the best signal. Then the obtained vectors U and V are used to reconstruct the Hankel matrix.

4. PERFORMANCE EVALUATION

The proposed signal noise reduction approach was implemented using Matlab 7.1. We have used two measures to evaluate its performance. The results are then compared with those obtained using the Wiener filter. The performance measures used in this comparison are briefly described below:

4.1. Mean Square Error

The Mean square error (MSE) metric is frequently used in signal processing and is defined as follows:

$$MSE = \frac{1}{L}\sum_{I=1}^{L}(S_{original}(i) - S_{denoised}(i))^2 \tag{13}$$

Here $S_{original}$ is the original signal and $S_{denoised}$ is the de-noised signal. The smaller MSE value, represents the better noise reduction algorithm.

4.2. Signal to Noise Ratio

The signal to noise ratio (SNR) is a well known measure in signal processing. It is defined as below:

$$(SNR)_{db} = \frac{Signal\ Power}{Noise\ Power} \tag{14}$$

This criterion indicates that how the noise was degraded in the enhanced signal, the larger SNR value represents the enhanced signal is closer to the original.

5. EXPERIMENTAL RESULTS

To evaluate the performance of the above-described algorithm, it applied on Multi-component and FM signals. Gaussian noise is added to these signals with SNR rate from 1 to 6. Then, noise in each signal is reduced separately using Wiener filter and the proposed method. These results are shown in Figures 1,2.

To statistically compare performance of the two approaches, these experiments have been repeated 100 times and the results are evaluated using MSE and SNR criteria and then, the average of the obtained results are trooped in the Figures 3,4,5 and 6. Figures 3 and 4 show SNR values for these two methods in multi-component and FM signals, respectively and in Figures 5 and 6, MSE values for these methods are presented.

These values show the high performance of proposed approach in signal de-noising. In addition, the proposed approach has a higher speed in comparison to Wiener filter. In these experiments SNR criteria is applied for defining the number of iterations.
The best results are obtained with iteration=10 $\Delta t = 0.1, k = 10$.

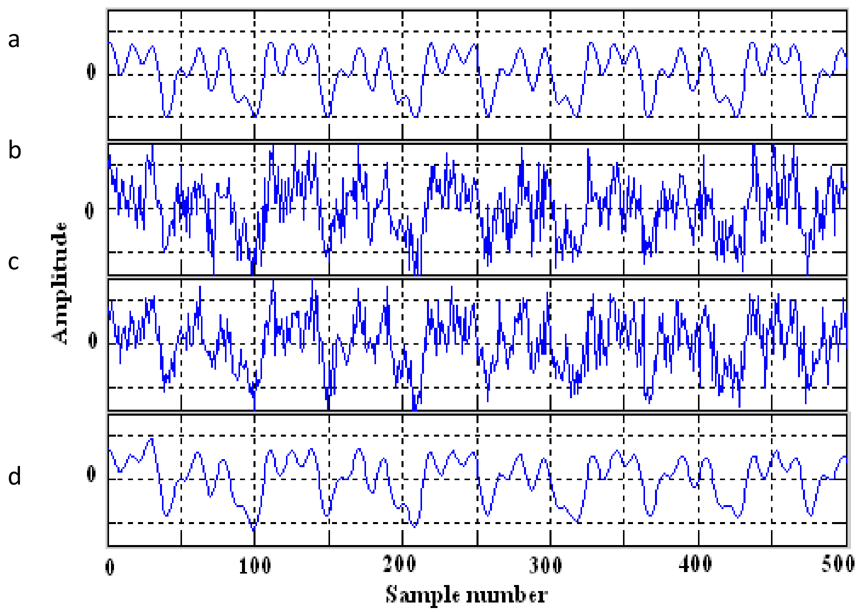

Figure1. Comparison of Wiener filter and the proposed method on a multi-component signal in noise reduction a)original signal b)noisy signal with Gaussian noise(SNR=1) c) result of the wiener filter d)result of the proposed method.

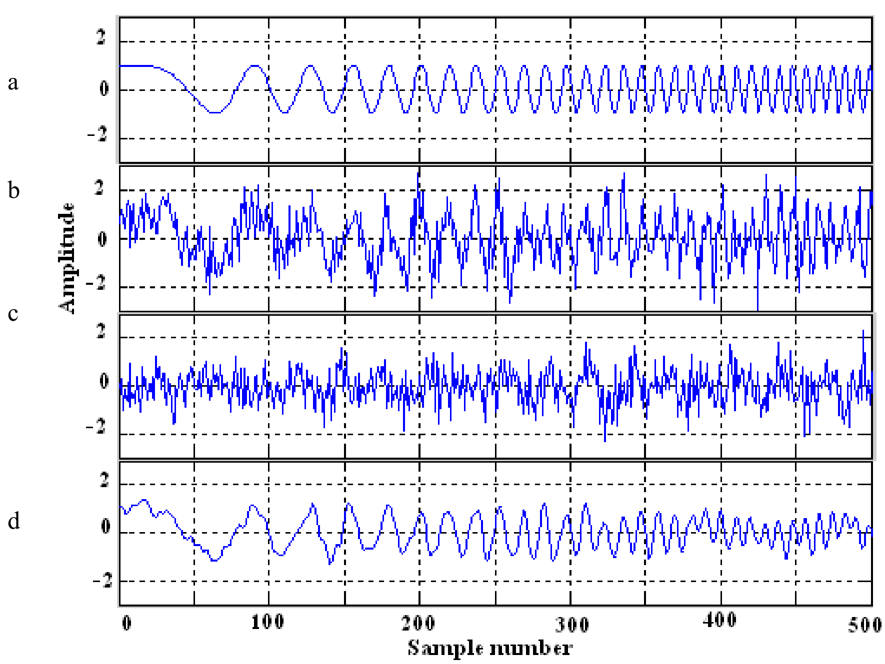

Figure2. Comparison of Wiener filter and the proposed method for a FM signal in noise reduction. a)original signal b)noisy signal with Gaussian noise(SNR=1) c) result of the wiener filter d)result of the proposed method.

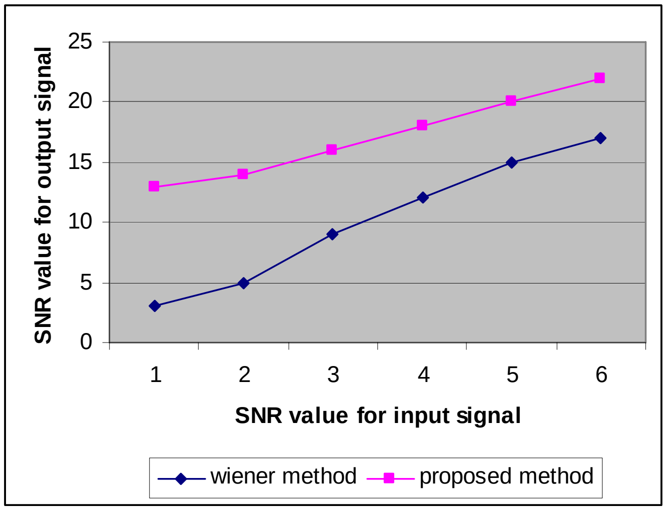

Figure 3. SNR values in Wiener method in comparison to proposed method in a multi-component signal de-noising.

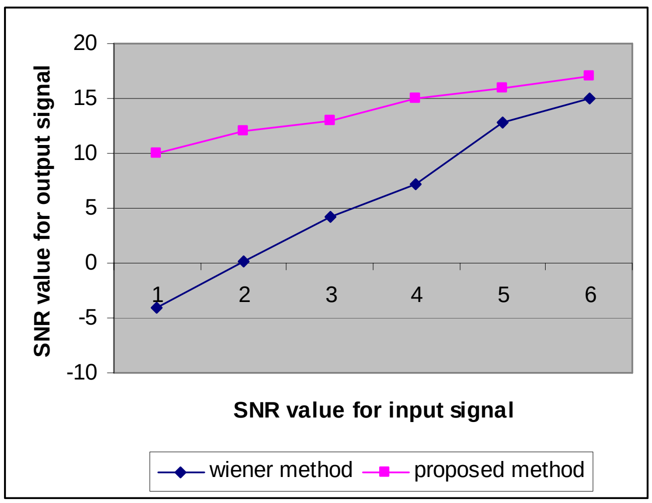

Figure 4. SNR values in Wiener method in comparison to proposed method in a FM signal de-noising.

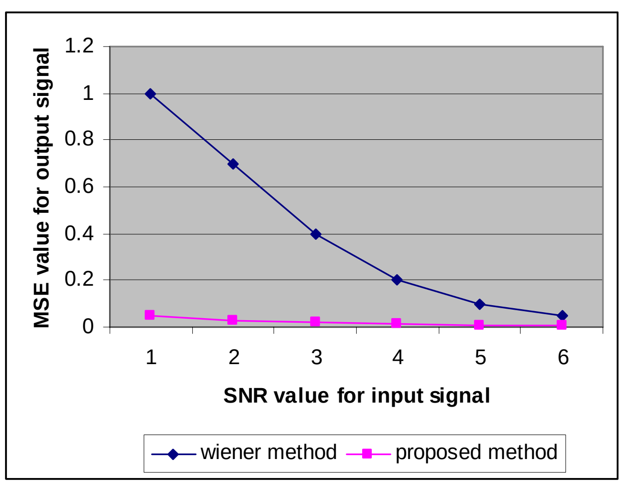

Figure 5. MSE values in Wiener method in comparison to proposed method in a Multi-component signal de-noising.

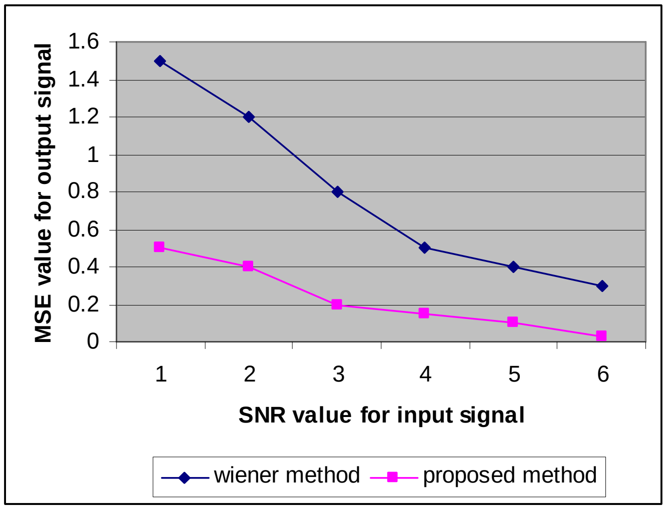

Figure 6. MSE variations in Wiener method in comparison to proposed method in a FM signal de-noising.

6. CONCLUSIONS

In this study, a new approach was introduced for signal noise reduction. This approach is based on partial differential equations (PDEs). In this approach, at first the signal is represented in Hankel matrix and then the SVD is used for Space division. To reduce the effect of noise from the singular vectors the PDE is applied on the singular vectors of the matrix representing the signal. This method has been compared with Wiener filter. The performance of these methods is evaluated with MSE and SNR metrics. Results show that this method has good performance and high speed in signal noise reduction.

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Comparison of Different Machine Learning Algorithms for the Initialization of Student Knowledge Level in a Learner Model-Based Adaptive E-Learning System

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Abstract:
Web-based learning systems give students the freedom to determine what to study based on each individual student's learning goals. These systems support students in constructing their own knowledge for solving problems at hand. However, in the absence of instructors, students often need to be supported as they learn in ways that are tailored to suit a specific student. Adaptive web-based learning systems are suited to such situations. In order for an adaptive learning system to be able to provide learning support, it needs to build a model of each individual student and then to use the attribute values for each student as stored in the student model to determining the kind of learning support that is suitable for each student. Examples of such attributes are student knowledge level, learning styles, student errors committed during learning, the student's program of study, gender and number of programming languages learned by the student of programming. There are two important issues about the use of student models. Firstly, how to initialize the attributes in the student models and secondly, how to update the attribute values of the student model as students interact with the learning system. With regard to initialization of student models, one of the approaches used is to input into a machine learning algorithm attribute values of students who are already using the system and who are similar (hence called neighbors) to the student whose model is being initialized. The algorithm will use these values to predict initial values for the attributes of a new student. Similarity among students is often expressed as the distance from one student to another. This distance is often determined using a heterogeneous function of Euclidean and Overlap measures (HOEM). This paper reports the results of an investigation on how HOEM compares to two different variations of Value Difference Metric (VDM) combined with the Euclidean measure (HVDM) using different numbers of neighbors. An adaptive web-based learning system teaching object oriented programming was used. HOEM was found to be more accurate than the two variations of HVDM.

Categories and Subject Descriptions: H.5.2 [**Information Interfaces and Presentation**]: User Interfaces – User Centered Design; H.5.4 [**Information Interfaces and Presentation**]: Hypertext/Hypermedia-Navigation, User issues; I.2.6 [**Artificial Intelligence**]: Learning – Concept learning; Induction; K.3.1 [**Computers and Education**]: Computer Uses in Education – Distance Learning, Computer Assisted Instruction (CAI)
General Terms: Algorithms, Human Factors, Experimentation, Measurement
Additional Key Words: Learner modeling, initialization, web-based learning, nearest neighbors, overlap measure, knowledge level, object oriented programming, value difference metric.

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